1

a) $z(2,0) = 2$

$$z_x = e^{xy} + x \cdot y\, e^{xy}$$

$$z_x(2,0) = e^0 + 0 = 1$$

$$z_y = x \cdot x e^{xy}$$

$$z_y(2,0) = 4\, e^0 = 4$$

Eq. plano tangente:

$$z = \cancel{2} + 1(x - \cancel{2}) + 4(y - 0)$$

$$z = x + 4y$$

b) $2.1 e^{0.21} = 2.1 e^{2.1 \times 0.1}$

$$\approx 2.1 + 4 \times 0.1$$

$$\approx 2.5$$

2

a) $u = x^2 - y^3$, $v = 3xy$

$$\frac{\partial w}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

$$= -3y^2 \frac{\partial f}{\partial u} + 3x \frac{\partial f}{\partial v}$$

b) $\dfrac{\partial^2 w}{\partial y \partial x} = \dfrac{\partial^2 w}{\partial x \partial y}$

$$= \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(-3y^2\frac{\partial f}{\partial u} + 3x\frac{\partial f}{\partial v}\right)$$

$$= -3y^2\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u}\right) +$$

$$+ 3\frac{\partial f}{\partial v} + 3x\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial v}\right)$$

$$= -3y^2\left[\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\right)\frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial u}\right)\frac{\partial v}{\partial x}\right]$$

$$+ 3\frac{\partial f}{\partial v}$$

$$+ 3x\left[\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial v}\right)\frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial v}\right)\frac{\partial v}{\partial x}\right]$$

$$= -3y^2\left[2x\frac{\partial^2 f}{\partial u^2} + 3y\frac{\partial^2 f}{\partial v\,\partial u}\right]$$

$$+ 3x\left[2x\frac{\partial^2 f}{\partial u\,\partial v} + 3y\frac{\partial^2 f}{\partial v^2}\right]$$

$$+ 3\frac{\partial f}{\partial v}$$

3

a) $\vec{v} = (0,0) - (-1,2) = (1,-2)$

$\|\vec{v}\| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{1}{\sqrt{5}}(1,-2)$

$z_x = y \Rightarrow z_x(-1,2) = 2$

$z_y = x \Rightarrow z_y(-1,2) = -1$

$\nabla z(-1,2) = (2,-1)$

taxa de variação:

$D_{\vec{u}} z(-1,2) = \nabla z(-1,2) \cdot \vec{u}$

$= (2,-1) \cdot \frac{1}{\sqrt{5}}(1,-2)$

$= \frac{1}{\sqrt{5}}(2+2)$

$$= \frac{4}{\sqrt{5}}$$

Como $D_{\vec{u}}\, z(-1,2) > 0$, a profundidade está a aumentar.

b) $\nabla z(-1,2) = (2,-1)$

4

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Leftrightarrow \begin{cases} e^x(y^2 - x^2) + e^x(-2x) = 0 \\ e^x 2y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \cancel{e^x}(y^2 - x^2 - 2x) = 0 \\ y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x(x+2)=0 \\ y=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=0 \\ y=0 \end{cases} \vee \begin{cases} x=-2 \\ y=0 \end{cases}$$

Pontos críticos $= \{(0,0), (-2,0)\}$

$$f_{xx} = \left(e^{x}(y^2-x^2-2x) \right)'_x$$

$$= e^{x}(y^2-x^2-2x-2x-2)$$

$$= e^{x}(y^2-x^2-4x-2)$$

$$f_{yx} = \left(e^{x}2y\right)'_x = 2ye^{x}$$

$$f_{yy} = \left(e^x 2y\right)'_y = 2e^x$$

<u>Ponto crítico (0,0):</u>

$$H(0,0) = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$$

$D = -4 < 0 \Rightarrow (0,0)$ é ponto de sela

<u>Ponto crítico (-2,0)</u>

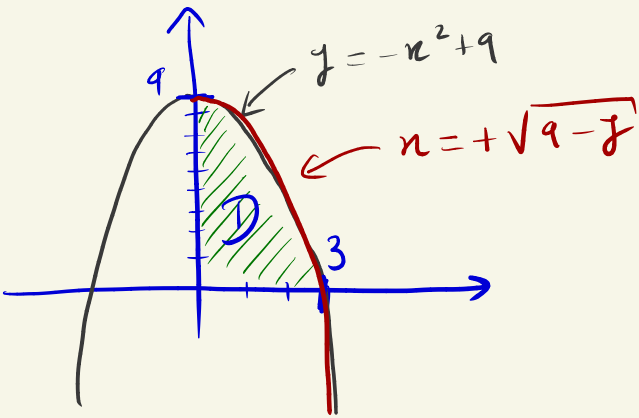
$$H(-2,0) = \begin{bmatrix} -2e^{-2} & 0 \\ 0 & 2e^{-2} \end{bmatrix}$$

$D = -4e^{-4} < 0 \Rightarrow (0,0)$ é ponto de sela

5

a) $D: \; 0 \leq x \leq 3$

$0 \leq y \leq -x^2+9$



b) $y=-x^2+9 \iff x^2 = 9-y$

$\iff x = \pm\sqrt{9-y}$

$D: \; 0 \leq y \leq 9$

$0 \leq x \leq +\sqrt{9-y}$

$$\int_0^3 \int_0^{-x^2+9} x y^2 \, dy \, dx = \int_0^9 \int_0^{\sqrt{9-y}} x y^2 \, dx \, dy$$

c)

$$\int_0^3 \int_0^{-x^2+9} x y^2 \, dy \, dx =$$

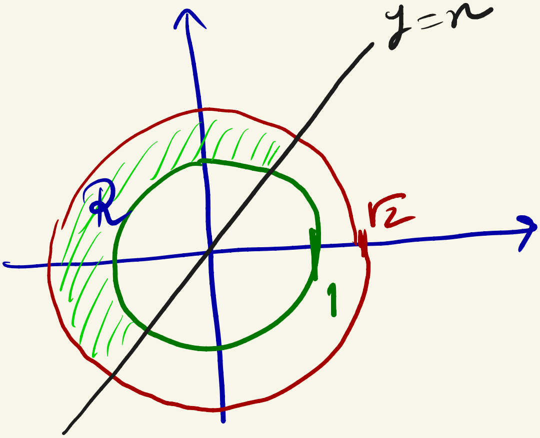
$$= \int_0^3 x \left[\frac{y^3}{3} \right]_0^{-x^2+9} dx$$

$$= \frac{1}{-2 \times 3} \int_0^3 -2x \left(-x^2+9\right)^3 dx$$

$$= -\frac{1}{6} \left[\frac{\left(-x^2+9\right)^4}{4} \right]_0^3$$

$$= -\frac{1}{24} \left(0 - 9^4\right) = \frac{9^4}{24}$$

6



$R: \quad 1 \leq r \leq \sqrt{2}$

$\frac{\pi}{4} \leq \theta \leq \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

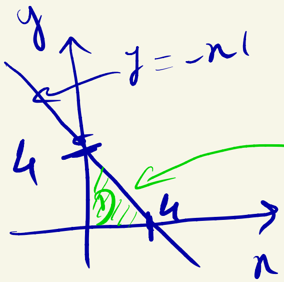
7 X = tempo de espera do cliente #1

Y = " " " " #2

Como X, Y são independentes

$$f_{X,Y}(x,y) = f_X(x) f_Y(y) = \begin{cases} \frac{1}{2} e^{-x/2} \frac{1}{2} e^{-y/2}, & x,y \geq 0 \\ 0, & \text{caso contrário} \end{cases}$$

$$P(X+Y \le 4) = \iint_{\{x+y \le 4\}} f_{X,Y}(x,y)\,dx\,dy$$



$$= \iint_{\{x+y\le 4,\ x,y \ge 0\}} f_{X,Y}(x,y)\,dx\,dy$$

D: $0 \le x \le 4$

$0 \le y \le -x+4$

$$= \int_0^4 \int_0^{-x+} \frac{1}{2}e^{-x/2}\,\frac{1}{2}e^{-y/2}\,dy\,dx$$

$$= -\int_0^4 \frac{1}{2}e^{-x/2}\left[e^{-y/2}\right]_0^{-x+4}$$

$$= -\frac{1}{2}\int_0^4 e^{-x/2}\left(e^{x/2-2} - 1\right)dx$$

$$= -\frac{1}{2}\int_0^4 e^{-2} - e^{-x/2}\, dx$$

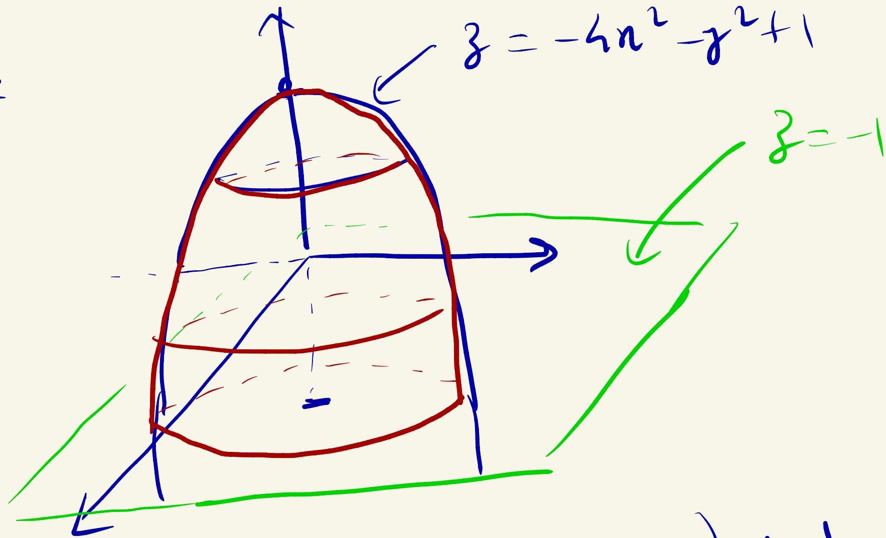
$$= -\frac{1}{2}\left\{ 4e^{-2} - \left[\frac{e^{-x/2}}{-1/2}\right]_0^4 \right\}$$

$$= -\frac{1}{2}\left\{ 4e^{-2} + 2\left(e^{-2} - 1\right) \right\}$$

$$= -\frac{1}{2}\left\{ 6e^{-2} - 2 \right\}$$

$$= 1 - 3e^{-2} \approx 0.59$$

8



$$\text{Vol} = \iint_D \left(-4x^2 - y^2 + 1 - (-1)\right) dx\,dy$$

$D: \quad -4x^2 - y^2 + 1 = -1 \Leftrightarrow$

$\Leftrightarrow 4x^2 + y^2 = 2$

$\Leftrightarrow (2x)^2 + y^2 = 2$

$\Leftrightarrow u^2 + v^2 = 2$

para $\begin{cases} u = 2x \\ v = y \end{cases} \Leftrightarrow \begin{cases} x = u/2 \\ y = v \end{cases}$

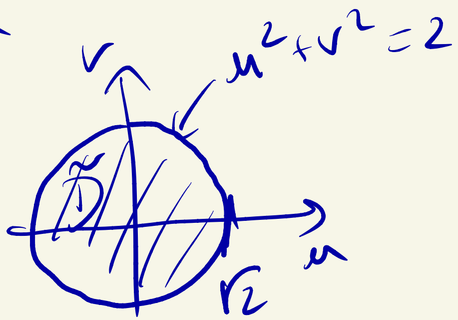
$$\frac{\partial(x,y)}{\partial(u,v)} = \det \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} = 1/2$$

Donde $\boxed{dx\,dy = \frac{1}{2}\,du\,dv}$ e

$$\text{Vol} = \iint_D -(2x)^2 - y^2 + 2 \;dx\,dy$$

$$= \iint_{\tilde{D}} \left(-u^2 - v^2 + 2\right)\frac{1}{2}\,du\,dv$$

onde



$\tilde{D}$: $0 \leq \theta \leq 2\pi$

$0 \leq r \leq \sqrt{2}$

$$\text{Vol} = \frac{1}{2}\int_0^{2\pi}\int_0^{\sqrt{2}} \left(-r^2+2\right) r \, dr \, d\theta$$

$$= \frac{\cancel{2}\pi}{\cancel{2}} \left[-\frac{r^4}{4} + r^2\right]_0^{\sqrt{2}}$$

$$= \pi \left(2 - 1\right) = \pi$$